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GAS AND PARTICLE MOTION IN JETS IN FLUIDIZED BEDS

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Abstract—Entrainment of solid particles by gas jets discharged downwards through slotted nozzles into bubble-free beds of fluidized particles is considered. The gas flow in the jet is calculated for irrotational flow, using a correlation established previously for slot opening as a function of operating variables. The momentum boundary layer thickness and shear stress at the horizontal interface between jet and particles are then calculated by integral boundary layer analysis. The calculated shear stress distributions are consistent with measurements of the momentum of bed particles caused to saltate by the jet, and explain the dependence of particle movement on the various operating variables. The results provide a direct confirmation of a hypothesis due to Owen on the mechanism of saltation.

INTRODUCTION

The present study is part of a comprehensive investigation on the quenching of flames in fluidized beds, and on possible applications to production of acetylene and other unsaturated hydrocarbons from methane-rich flames (Russo & Massimilla 1972). A jet of hot gas introduced into a fluidized bed characteristically experiences a sharp temperature drop, and much of the rapid quenching appears to result from heat transfer to particles entrained by the gas. Therefore particle entrainment rates are of considerable practical interest.

As in earlier studies reported by Massimilla & Russo (1972, 1973), slotted rectangular nozzles immersed in fluidized beds have been considered. The general configuration is shown in figure 1. The jet gas is introduced vertically downwards into the nozzle, passes out from the jet through slots on opposite sides, and then rises through the bed as bubbles. In the acetylene process, the bed is operated very close to minimum fluidization in order to maintain a high acetylene concentration in the exit gas (Russo & Massimilla 1972). This work therefore considers beds operated under conditions of bubble-free fluidization. It may also be noted that the jet outlet gas velocity is much lower than in other studies of horizontal gas jets (Merry 1971). Russo & Massimilla (1972) have given an analysis based on potential flow for the idealized case in which the slot opening is vanishingly small. The results showed that the fluidizing gas penetrates into the gas in the nozzle, but that the depth of penetration beyond the interface with the particulate phase is very small. Subsequently they developed a correlation for the mean open slot height (h), and reported observations of particle movement in the jet. In this work the potential flow analysis is extended to allow for finite slot opening, and an integral boundary layer analysis is used to calculate the flow pattern close to the interface. The results are related to observed particle movement via a theory of saltation proposed by Owen (1964).

EXPERIMENTAL DETAILS AND OBSERVATIONS

Experiments were carried out in a column of semi-circular cross-section with 140 mm i.d. (*cf.* figure 1), described in detail by Massimilla *et al.* (1972). Two rectangular nozzles of the type sketched in figure 2 were used, with dimensions given in table 1. Each nozzle was glued to a window of optical glass in the flat face of the column, so that the window itself formed one face of the nozzle. Slots were cut in the two vertical faces normal to the window, and a horizontal porous plate was soldered inside each nozzle to ensure even distribution of the jet gas. Most of the experiments considered here were carried out using Argon as the jet gas, with some supplementary experiments using Freon-12.

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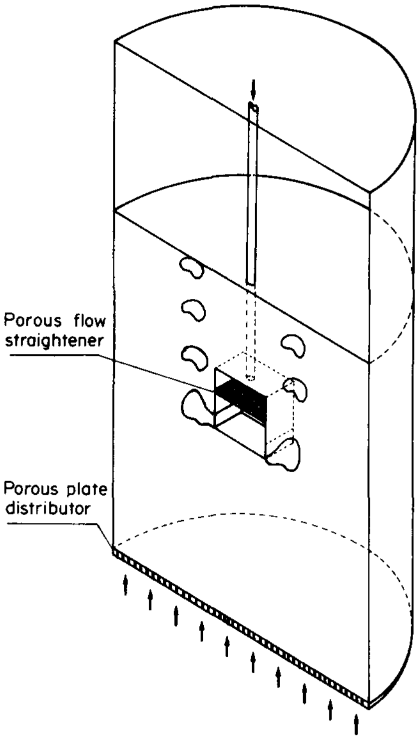

Figure 1. Sketch of the experimental set-up. (Not to scale).

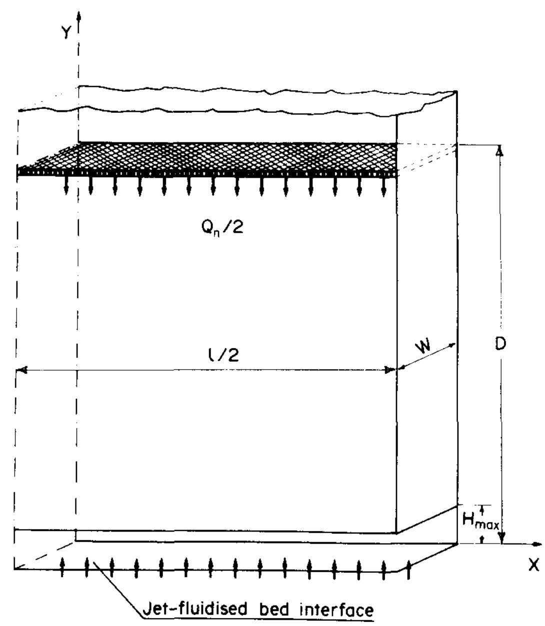

Figure 2. Rectangular slotted jet (schematic).

Table 1. Details of experimental systems

		Jet dimensions			
Nozzle assembly		Width, l (cm)	Thickness, w (cm)		Height, D (cm)
A		2.7	1.6		2.5
B		5.4	3.2		5.0
			Bed particles		
	Density (g/cm^3)	Range of d_p (μm)	Surface mean d_p (μm)	Min. fluidizing velocity (N_2) (cm/sec)	Critical shear stress ($dynes/cm^2$)
Fluid cracking catalyst	1.0	40–175	60	0.7	0.038
Ballotini	2.7	125–170	140	7	0.24

Two types of bed particle were used, with properties summarized in table 1. Nitrogen was used as the fluidizing gas throughout. With the cracking catalyst, no spontaneous bubbling was observed with superficial velocities up to three times that of minimum fluidization. As found by Massimilla & Russo (1973), the fluidizing gas velocity had no detectable effect on slot opening or particle behaviour in this range. For the glass ballotini, bubbles formed as soon as the minimum fluidization velocity was exceeded, and all experiments were carried out with the bed at incipient fluidization.

The particle movement in the vicinity of the jet/bed interface was recorded by cine photography, at about 700 frames per second. As in the observations of Massimilla & Russo (1973), films taken with the camera oriented at 45° to the window provided a record of the rate of arrival of particles at the interface, and of the velocities of individual particles set in motion by the jet gas. In addition, some films were taken with the camera axis normal to the window, to record the trajectories of individual particles. Full details of the observations of particle movement are given by Höfelsauer-Saggese (1972). Vigorous particle movement was restricted to a region of the order of a few particle diameters in thickness at the jet/bed interface. Figure 3

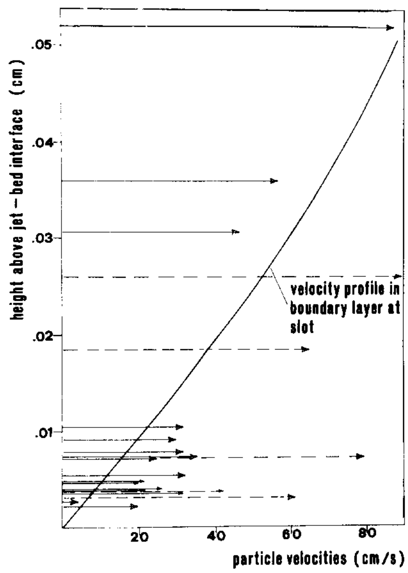

Figure 3. Particle velocities close to slot; nozzle A, argon–FCC, $Re = 1000$.

shows a typical set of measurements of particle velocities within 2 mm from an exit slot. Most moving particles remained within typically three particle diameters from the interface, and moved with mean velocity of the order of 10% of the mean gas velocity in the slot. Occasional particles rose up to ten diameters from the interface, and then acquired much higher velocities, as shown in figure 3. These particles normally appeared to originate from wave-like disturbances on the interface, or to be projected upwards by particle collisions. The high velocity was maintained as the particle settled back towards the interface, accounting for the abnormally large velocity sometimes observed for particles in the lower part of the moving layer (dashed lines in figure 3). The motion of the particles in the bed was symmetrical about the centre-line of the jet, provided that the flowrate of jet gas Q_n was less than a critical value discussed by Massimilla & Russo (1973).

Observations of the rate of particle arrival in the mobile layer and of the mean velocity of moving particles as functions of position in the jet were also reported in the earlier study. The qualitative conclusions were confirmed and extended by the present work. It is convenient to describe the particle movement by two ratios:

$$F_N = \frac{\text{No. of particles crossing section per unit time}}{\text{No. of particles leaving slot per unit time}}, \qquad [1]$$

$$F_{\bar{v}} = \frac{\text{mean velocity of particles crossing section}}{\text{mean velocity of particles leaving slot}}. \qquad [2]$$

Figure 4 shows measurements of F_N and $F_{\bar{v}}$ for experiments with argon as the jet gas. The following qualitative conclusions are apparent:

1. With cracking catalyst in the smaller nozzle (A) at relatively high gas rate ($Re = 1000$ curve 1), most of the moving particles originate in the portion of the jet close to the centre-line, but the mean velocity increases over the whole width of the jet.

2. As the gas rate is reduced, the curves are displaced to the right (cf. curves 1 and 2); i.e. the region close to the exit slot predominates.

3. With glass beads (curve 3), the rise of both F_N and $F_{\bar{v}}$ is displaced towards the exit slot.

4. In the larger nozzle (B; curves 4 and 5), the portion of the jet close to the centre-line is much less active in promoting particle movement. This is particularly marked for glass beads

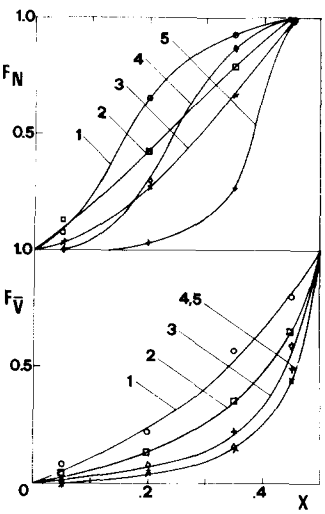

Figure 4. Cumulative frequency of particle emergence (F_N) and particle velocity ratio ($F_{\bar{v}}$) for various operating conditions:

Symbol	Curve	Nozzle	Gas	*Re*	Particles
○	1	A	Ar	1000	FCC
□	2	A	Ar	500	FCC
×	3	A	Ar	250–1500	Ballotini
◇	4	B	Ar	500–3000	FCC
+	5	B	Ar	500–3000	Ballotini

(curve 5), where virtually no particle movement is observed in the central half of the jet. For cracking catalyst, the jet dimensions appear to have more effect on $F_{\bar{v}}$ than on F_N.

Analogous results on the effect of particle characteristics and nozzle size were obtained in experiments with Freon-12. Similar velocities were required to cause comparable particle movement. Thus, because of the property differences between freon and argon, the Reynolds number for freon is greater by typically a factor of 5 to obtain values of F_N and $F_{\bar{v}}$ comparable to those in figure 4.

GAS FLOW IN JET

Figure 4 confirms the observations of Massimilla & Russo (1973) that F_N and $F_{\bar{v}}$ develop in different fashion across the interface. A simple particle balance shows that these two parameters should be equal if particle emergence results solely from renewal of the interface as it is "stretched" towards the outlet slot. Evidently some more complex phenomenon such as saltation is responsible for the observed motion.

Dimensional analysis indicates that the following dimensionless groups govern the observed phenomena:

1. Reynolds number:

$$Re = V_s l / \nu \qquad [3]$$

where V_s is the superficial velocity of gas in the jet of width l, and ν is the kinematic viscosity;

2. Froude number:

$$Fr = V_s^2 / gl \qquad [4]$$

where g is the gravitational constant;

3. Density ratio:

$$s = \rho_g / \rho_D \qquad [5]$$

where ρ_g is the gas density and ρ_D is the mean density of the dense phase of the fluidized bed. The Reynolds number arises from the momentum balance for the gas phase, while Fr and s arise from the momentum balance which determines conditions at the gas outlet from the nozzle. On the assumption, discussed below, that the dense phase properties only affect the gas flow by their effect on the slot opening, these groups suffice to describe the gas motion. However, even before particle motion is considered, it is clear that similarity in terms of a single group such as Re will not apply, and this is confirmed by the results in figure 4.

In addition to the parameters governing the gas motion, it is necessary to include parameters to describe the behaviour of particles in the jet. The bed is always close to minimum fluidization, and only phenomena at the interface between the gas jet and the fluidized bed are of interest here. Moreover, Massimilla & Russo (1973) found that F_N and $F_{\bar{v}}$ are insensitive to the fluidizing gas velocity even up to gas rates slightly in excess of minimum bubbling. Thus, a Reynolds number for the dense phase is not a relevant parameter. The analysis in the following section suggests that the essential parameter is the shear stress at the interface, τ. An appropriate dimensionless group is then the friction factor:

$$c_f = 2\tau / \rho_g V_s^2. \qquad [6]$$

The friction factor is a function of the dimensionless distance from the centerline of the jet, $X = x/l$, so that we may write

$$c_f = \phi(Re, Fr, s, X). \qquad [7]$$

Massimilla & Russo (1973) established that the slot opening for jets of the type used here are correlated well by a form used for predicting slot openings of bubble caps in gas/liquid contactors. In dimensionless form, the correlation is

$$H = \left(\frac{0.53}{k}\right)^{2/3} (sFr)^{1/3}, \qquad [8]$$

where H is the dimensionless slot opening (h/l), and k is an empirical orifice constant, that in this case equals 0.51. With this correlation, the separate parameters s and Fr can be replaced by their product, so that [7] contracts to

$$c_f = \phi(Re, sFr, X). \qquad [9]$$

It is shown below that the particle movement depends upon the distribution of c_f and on a critical value c_{fc} corresponding to the critical shear stress for initiation of saltation, τ_c. Thus

$$F_N = \phi_N(c_f, c_{fc}) = \phi_N(Re, sFr, X, c_{fc}), \qquad [10]$$

$$F_{\bar{v}} = \phi_{\bar{v}}(c_f, c_{fc}) = \phi_{\bar{v}}(Re, sFr, X, c_{fc}). \qquad [11]$$

The four groups in [10] and [11] are the minimum necessary to describe the jet–particle interaction. It is shown below that it is also necessary to account for the mechanism which causes particles to move from the interface, although in the present state of knowledge it is not possible to do this explicitly. Equations [9]–[11] are the appropriate forms for scale-up of such a jet in a bubble-free fluidized bed. However, for theoretical analysis of the results in figures 3 and 4 the nozzle size will be used as a more immediate substitute for *Fr*, and the jet gas and bed particles will be referred to directly instead of via s or c_{fc}.

An irrotational flow solution for the bulk gas motion in the jet, coupled with a boundary layer analysis for the interface is suitable for predicting the relationship in [9]. Details of the analysis are given in the appendix. The irrotational flow was calculated by standard numerical solution for the dimensionless stream function, using the slot opening calculated from [8]. Figure 5 shows a typical flow pattern, calculated for argon and fluid cracking catalyst in nozzle A at $Re = 1000$ ($s = 3.32 \times 10^{-3}$, $Fr = 0.062$). The velocity profile in the outlet ES is also shown. For the subsequent boundary layer analysis, the relevant velocity profile is the dimensionless velocity $U(X)$ at $y = 0$. Figure 6 shows $U(X)$ for the conditions of figure 5, together with theoretical solutions for two limiting cases:

Curve 1: Flow of a gas jet through a nozzle with vanishingly small aperture (Russo & Massimilla 1972):

$$U(X) = \frac{\sin \pi(1-2X)}{1-\cos \pi(1-2X)}. \qquad [12]$$

Curve 2: Flow from an otherwise unbounded region through a slot of width $2H$ (Milne-Thomson 1972):

$$U(X) = \frac{1}{\pi\sqrt{(H^2 + (0.5 - X)^2)}}. \qquad [13]$$

As may be expected, [12] provides a good approximation to the numerical results close to the centre line, while [13] is approximated near the slot. The subsequent boundary layer analysis requires $U(X)$ as a continuous function with continuous first derivative. The numerical values were represented by piecewise approximating functions, given in the Appendix, fitted by a standard least-squares approach. Figure 6 shows the resulting profile for $U(X)$, and confirms that the fit to the numerical values is close (normally within 2% of U). As Re decreases, the point of inflection in $U(X)$ moves towards the slot; i.e. the "slot region" of the jet, in which [13] is approximated, becomes relatively smaller. Similar trends result from increasing the jet dimensions at constant Re.

The shear stress at the interface was evaluated from $U(X)$ by a von Karman-Pohlhausen integral momentum balance analysis, outlined in the appendix. Since particle velocities at the interface are a small fraction of the gas potential flow velocity (see below), the interface was treated as a rigid boundary. The boundary layer Reynolds number in all cases remained well

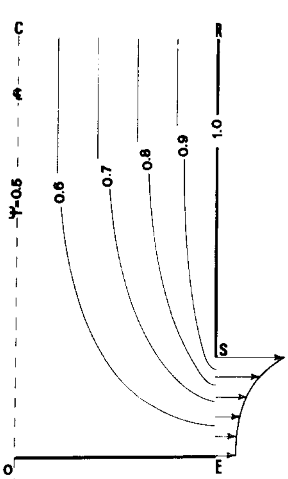

Figure 5. Irrotational flow solution: nozzle A, argon, $Re = 1000$.

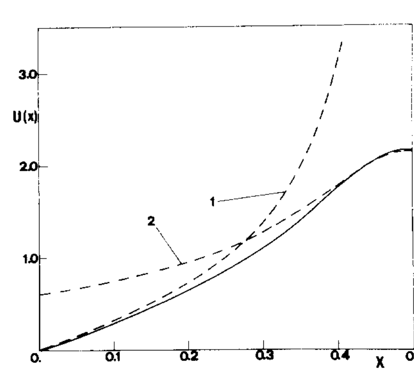

Figure 6. Irrotational flow velocity at interface $U(X)$, nozzle A, argon, $Re = 1000$. --- Limiting theoretical results [13] and [14]; ——— fitted curve [17] and [18].

below the critical value (approximately 360) for transition to turbulence. Therefore friction factors were evaluated assuming laminar flow in the boundary layer. Figures 7 and 8 show the resulting predictions for the dimensionless boundary layer momentum thickness δ_2 and for the dimensional shear stress, τ. For a given nozzle and jet gas, the effect of increasing *Re* is to decrease δ_2 and increase τ, as expected. It is interesting to note that the maximum value of τ is only weakly dependent on *Re* and jet dimensions. This results from the compensating effect of increased open slot height at higher *Re*. However, the form of $\tau(x)$ is very strongly dependent on *Re* and on the jet width l: increasing *Re* and decreasing l cause the shear stress to rise more rapidly in the central portion of the jet. It is shown below that this gives rise to increased particle movement in this region.

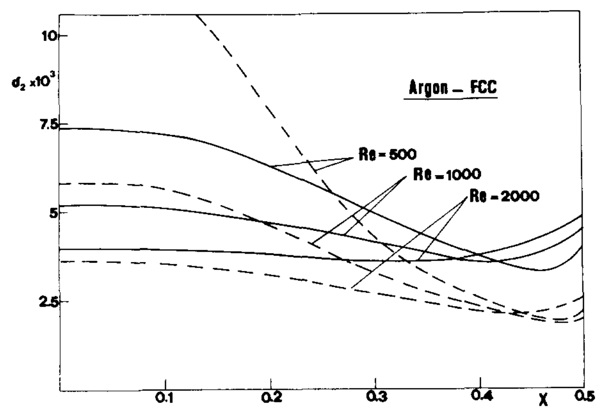

Figure 7. Momentum boundary layer thickness. ——— Nozzle A; --- nozzle B.

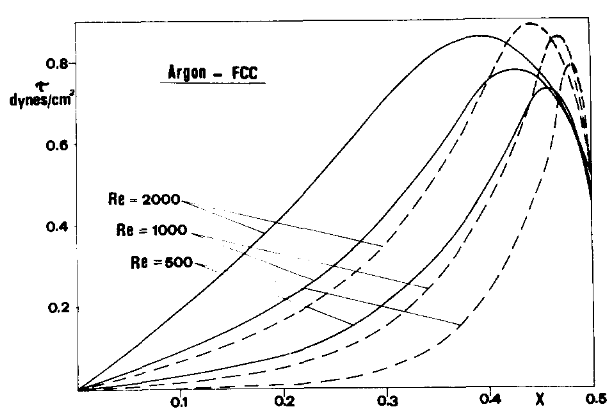

Figure 8. Interfacial shear stress. ——— Nozzle A; ---- nozzle B.

MECHANICS OF PARTICLE SALTATION

The general particle behaviour described earlier shows all the characteristics of saltation, described for example by Bagnold (1941). Saltation occurs when a fluid flows over a mass of solid particles under conditions such that the shear stress is sufficient to maintain movement of particles close to the interface, but is insufficient to carry the particles into suspension. Generally particles appear to enter into suspension if the shear stress is of order $\rho_p g d$, where ρ_p and d are respectively the particle density and diameter (Owen 1964). The corresponding numerical values for τ are an order of magnitude larger than the values predicted by the boundary layer analysis. Also the lift on an individual particle, estimated by the expression due to Saffman (1965, 1968) is insufficient to cause particles to rise from the interface. Thus theoretical considerations confirm the observations that the particle movement should be treated as saltation rather than suspension.

It may also be noted that measurements of the rheological properties of similar particulate materials (Rietema 1974) indicate a yield stress for bulk movement of the order of 10–20 dynes/cm^2. Thus shear on the interface in the gas jet is too small to set up bulk circulation. This is also consistent with the observation that particles are conveyed out of the jet by saltation, and replaced by fresh particles which rise to the interface, to be conveyed in turn to the slots.

The most complete theoretical treatment of saltation appears to be that due to Owen (1964). Owen suggested that part of the shear between the fluid and the interface is required to maintain the surface in a mobile condition, while the remainder is transmitted to the momentum of saltating particles. This mechanism explains how the particle concentration and velocity in the saltating layer adjust to an equilibrium level. Owen applied this hypothesis to saltation under conditions where τ is constant over the interface by considering the effect of the particles on the fluid velocity. Similar treatment is not applicable to the present experiments, because of the variations in τ and because the scale of the jets used is insufficient for "equilibrium" particle concentrations and velocities to be established. The approach taken here is to relate the shear stress predicted by the boundary layer analysis to the particle movement using Owen's basic hypothesis but neglecting the effect of particle movement on the gas. The observed particle velocities confirm that this is a reasonable approximation. Figure 3 shows that the velocities of particles moving almost parallel to the interface are close to the predicted fluid velocity profile. The agreement is poorest close to the interface, where the fluid velocity is assumed to fall to zero. However, it may be noted that, if the theoretical velocity profile is displaced downwards by a distance of the order of one particle diameter, the agreement becomes close even in this region. The practical definition of the location of the static interface is in any case not clear, so that the particle velocities are not inconsistent with the analysis of fluid motion. These measurements show that momentum transfer between gas and particles in fact occurs over a finite layer of the order of a few particle diameters in thickness. On the other hand, the boundary layer models assumes momentum transfer at one plane only. The essential parameter is the total momentum transfer, which is insensitive to this assumption. It may also be noted that the maximum observed height of rise of a saltating particle was about half the boundary layer thickness.

Figure 3 also shows that particles which fall back towards the interface have a velocity much larger than the local average. Owen assumed that, in "equilibrium" saltation, particles lose their horizontal momentum on impact with the surface. This phenomenon was not observed in the present experiments, presumably because, as noted above, "equilibrium" saltation is not established. Instead, virtually all particles entering into saltation were observed to move across the jet to the exit slot. Owen's hypothesis therefore implies that all the shear stress above that required to maintain a mobile interface is conserved as particle momentum. According to this hypothesis, the total flux of particle momentum per unit jet width across a section at x from the centre-line is given by

$$M = \int_{x_c}^{x} (\tau - \tau_c)\,\mathrm{d}x, \qquad [14]$$

where τ_c is the shear stress required to maintain the interface in motion and x_c is the distance from the centre-line at which $\tau = \tau_c$. Direct measurements of M are not available, due to the difficulty of obtaining absolute measurements of particle flux. It is therefore convenient to use the ratio:

$$F_M = \frac{\text{particle momentum crossing section in jet}}{\text{particle momentum in slot}},$$
$$= F_N F_{\bar{V}} \qquad [15]$$

where F_N and $F_{\bar{V}}$ have been determined as noted earlier. From [14], F_M may be written in the form of [10] and [11] as:

$$F_M = \int_{X_c}^{X} (c_f - c_{fc})\,dX \Big/ \int_{X_c}^{0.5} (c_f - c_{fc})dX. \qquad [16]$$

From experiments of Bagnold (1941) and Chepil (1945), τ_c for a fixed bed of particles is given closely by

$$\tau_c = 0.0064\rho_p g d. \qquad [17]$$

From the observation that the fluidizing gas velocity has no detectable effect on the motion of particles in the jet, it is concluded that fluidization conditions have little effect on τ_c. This may also be expected since the jet gas suppresses the flow of fluidizing gas through the particles close to the interface (Massimilla & Russo 1973). Therefore [17] is used to estimate τ_c for the present system; corresponding values are given in table 1. Equation [16] can then be used to calculate F_M by numerical integration using values of c_f and X_c from the boundary layer analysis.

Figure 9 shows the corresponding predictions for argon in nozzle A. For the catalyst particles, F_M appears to rise more rapidly initially than predicted by [16]. However, this may be ascribed to the fact that a distribution of particle sizes is present; τ_c and X_c were evaluated for the mean particle diameter, whereas smaller particles will saltate sooner. It is more significant that the theory predicts the correct effect of Reynolds number. For glass beads, results are only available as mean values over a range of Re, but the theory clearly predicts the correct

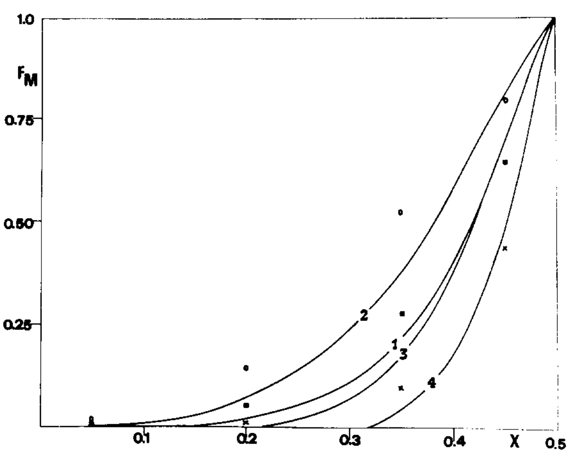

Figure 9. Particle momentum ratio in nozzle A: argon. Experimental results: □: FCC, $Re = 500$; ○: FCC, $Re = 1000$; ×: Ballotini, $Re = 250$–1500. Theoretical predictions from [27]: 1. FCC, $Re = 500$; 2. FCC, $Re = 1000$; 3. Ballotini, $Re = 500$; 4. Ballotini, $Re = 1000$.

dependence of F_M on particle characteristics. Corresponding measurements and predictions for nozzle B are shown in figure 10. Again the agreement is good, so that Owen's approach predicts the correct dependence on jet dimensions. The predicted values of X_c are also consistent with the observation that saltation of glass beads in nozzle B is confined to roughly the outer half of the nozzle.

Figure 10 also shows results obtained with Freon-12 in nozzle A. Agreement is not exact, since the experimental results cover a range of Re. However, the model shows why higher Re must be used with freon than with argon, and again predicts the correct effect of particle characteristics. There is no indication of anomalous behaviour resulting from the tendency of freon to cause particle aggregation, observed by Massimilla & Russo (1973). This confirms that the fluidizing nitrogen penetrates through the interface as predicted by Russo & Massimilla (1972), and thus prevents contact between the jet gas and the particles in and below the interface.

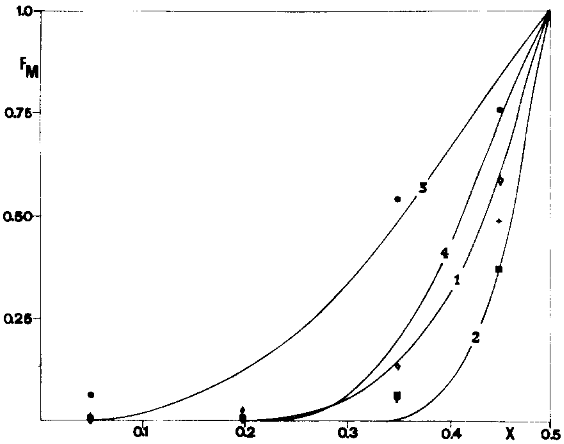

Figure 10. Particle momentum ratio: Experimental results: ◇: Ar–FCC, nozzle B, $Re = 500–3000$; +: Ar–Ballotini, nozzle B, $Re = 500–3000$; ●: freon–FCC, nozzle A, $Re = 1200–5000$; ■: freon–Ballotini, nozzle A, $Re = 1600–6000$. Theoretical predictions from [27]: 1. Ar–FCC, nozzle B, $Re = 1000$; 2. Ar–Ballotini, nozzle B, $Re = 1000$; 3. freon–FCC, nozzle A, $Re = 5000$; 4. freon–Ballotini, nozzle A, $Re = 5000$.

DISCUSSION

Since momentum transfer from jet fluid to particles appears to determine the value of the product $F_N F_{\bar{v}}$, the variations of F_N and $F_{\bar{v}}$ can be explained qualitatively in terms of the observed particle behaviour. Particle ejection, leading to relatively high fast trajectories, was observed to occur in regions of high τ when substantial particle movement is already established; i.e. in the vicinity of the exit slot at high gas rates, particularly with the fine cracking catalyst. When this occurs, shear is transmitted to the saltating particles as a rise in mean velocity rather than in emergency of fresh particles, so that $F_{\bar{v}}$ then rises more rapidly than F_N. Therefore F_N must rise towards unity in the inner part of the jet, explaining the different forms of the curves of F_N and $F_{\bar{v}}$ (cf. figure 4). If the saltating particles are restricted to a layer of constant thickness in which the solids fraction is constant, a simple particle balance indicates that F_N and $F_{\bar{v}}$ should be equal. This is approximately true for glass beads in nozzle B (see figure 4) for which very little ejection of particles from the surface occurred. However, for other cases, a direct estimation of particle flux or mean particle velocity would have to follow from a detailed analysis of the processes by which particles are projected upwards from the interface. Such an analysis is not currently available. Bagnold (1941) found that, in steady saltation, the mass flux of particles is proportional to $(\tau/g)\sqrt{(\tau d/\rho_p)}$. However, it would be surprising if this result were applicable to the kind of non-equilibrium saltation considered here, and comparison of the curves of F_N and τ immediately confirms that Bagnold's law does not hold. The observations of particle velocity and the momentum transfer analysis suggest that a reasonable estimate for the mass flux of particles at the slot could be obtained by assuming that the mean particle velocity is approximately 20% of the mean gas velocity in the slot, and evaluating the total particle momentum by [25], giving

$$\text{Particle mass flux in slot per unit width} \doteq \frac{5}{V_s}\int_{x_c}^{l/2} (\tau - \tau_c)\,dx. \qquad [29]$$

However the accuracy of this estimate should be supported by wider experimental evidence.

The analysis developed here indicates some general guidelines for design and scale-up of horizontal nozzles in beds operated in the bubble-free range. For maximum contact between jet gas and particles, the interfacial shear should exceed τ_c as close as possible to the centre of the jet. This implies that a number of small jets are preferable to a few large nozzles (cf. figure 8). For dense or coarse particles (large τ_c), the central portion of the jet will be essentially inoperative since $\tau < \tau_c$. Any design modification which increases τ should improve contacting in the jet. Due to the number of dimensionless groups in [10] and [11], any empirical scale-up procedure based on one or two dimensionless parameters is unreliable. For large or dense particles, where saltation is restricted to the region close to the slots, the single most important parameter determining particle flux and velocity is the gas velocity in the slot. For light or fine particles, this parameter is not sufficient to ensure similarity between different jet sizes, because movement occurs over a larger proportion of the nozzle and thus depends on the distribution of τ. It may be noted that these general conclusions are unlikely to apply in bubbling fluidized beds, where particle motion at the jet interface is strongly influenced by bubble-induced agitation.

CONCLUSIONS

Observations of particle movement in downward-facing jets, reported here and in previous publications, are in qualitative and quantitative agreement with predictions based on Owen's hypothesis on the mechanism of particle saltation due to shear from a gas stream. This provides a direct confirmation of Owen's hypothesis, which does not appear to have been available previously. While it appears possible to estimate particle momentum flux in this way, the frequency or velocity of particles moving across the jet cannot be predicted separately at present. The theoretical analysis shows that empirical scale-up of jets of the type used here is unreliable, and that uncertainties in scale-up are greatest for large nozzles and fine particles.

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APPENDIX—ANALYSIS OF GAS FLOW

Irrotational flow solution

The irrotational flow pattern was calculated in terms of a dimensionless stream function by numerical integration of the governing equation:

$$\nabla^2 \psi = 0. \qquad [A1]$$

Appropriate boundary conditions, applied to the boundaries indicated in figure 5, are:

(a) $\psi = 0.5$ at the centre line OC and at the interface OE.
(b) $\psi = 1.0$ at the nozzle wall RS.
(c) $\partial\psi/\partial X = 0$ in the slot ES.
(d) $\psi = 0.5 + X$ far upstream on section CR.

Condition (c) was used in preference to prescribing the velocity profile in the slot. To ensure that CR was far enough upstream for its location to have no effect on flow in the vicinity of the interface, *ER* was taken as 30 times ES. Solutions were obtained by successive overrelaxation of the set of algebraic equations obtained by replacing equation A1 by its finite difference approximation to $0[(\Delta X)^2 + (\Delta Y)^2]$. Optimum overrelaxation factors were calculated as suggested by Frankel (1950).

To approximate the numerical values for $U(X)$ at $Y = 0$ by an analytic form, two matched expressions were used. For the portion between the stagnation point and the inflection point, the form was

$$U = a_1X + b_1Xc_1 \quad (c_1 > 2) \qquad \text{[A2]}$$

while the expression for the segment between the inflection point and the slot was:

$$U = a_2/[1 + b_2(1 - 2X)^{c_2}]^d. \qquad \text{[A3]}$$

These functions automatically satisfy the requirements:

$$U'' \equiv \mathrm{d}^2U/\mathrm{d}X^2 = 0 \quad \text{at} \quad X = 0 \qquad \text{[A4]}$$

$$U' \equiv \mathrm{d}U/\mathrm{d}X = 0 \quad \text{at} \quad X = 0.5. \qquad \text{[A5]}$$

The parameters in [A2] were fitted by a standard least squares approach. The numerical value of U at the slot ($X = 0.5$) was equated to a_2, and finally d was fitted by least squares with b_2 and c_2 selected to give continuity of U and U' at the inflection point.

In most cases 21 grid points were used to span X and 151 points were used along y in all cases. This normally gave sufficient points for fitting [A2] and [A3]. When the inflection point was very close to the centre line (high Re) or to the slot (low Re), 81 points were used to span X. This gave no significant improvement in the accuracy of integration of [A1], but was necessary to provide sufficient points for the least squares approximations.

Boundary layer solution

Assuming that the interface between jet gas and bed particles can be approximated as a rigid surface, the von Karman–Pohlhausen integral momentum balance can be written:

$$\frac{\mathrm{d}\delta_2}{\mathrm{d}X} + \frac{U'}{U}\delta_2\left(2 + \frac{\delta_1}{\delta_2}\right) = \frac{c_f}{2U^2} \qquad \text{[A6]}$$

where δ_1 and δ_2 are respectively the dimensionless displacement thickness and momentum thickness. Making use of the approximation of Walz (see Schlichting 1968), [A6] takes the simpler form:

$$\frac{\mathrm{d}\delta_2^2}{\mathrm{d}X} = \frac{0.470}{Re}\frac{1}{U} - 6\frac{U'}{U}\delta_2^2. \qquad \text{[A7]}$$

Equation [A7] was integrated numerically with initial condition:

$$U\frac{\mathrm{d}\delta_2^{\,2}}{\mathrm{d}X}=0 \quad \text{at} \quad X=0. \qquad [A8]$$

Since the value of the boundary layer Reynolds number was always less than 300, the friction factor was calculated assuming laminar flow as

$$c_f=\frac{4U}{Re\delta}+\frac{UU'\delta}{3} \qquad [A9]$$

where the dimensionless boundary layer thickness δ is related to δ_2 by (Schlichting 1968)

$$\delta_2=\frac{1}{63}\left[\frac{37\delta}{5}-\frac{ReU'\delta^3}{15}-\frac{(ReU')^2\delta^5}{144}\right]. \qquad [A10]$$